# Integration of supervised and reinforcement learning for train wheel wear management

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## Abstract

Train wheel wear significantly impacts railway operations in terms of safety, passenger comfort, maintenance, reliability, and operational efficiency. This study focuses on managing train wheel wear consisting of changes in tread wear, flange height, and flange thickness. Regular wheel profile measurements are essential but are often limited by constraints such as heavy traffic, budget restrictions, time, and remote asset locations. These limitations challenge traditional maintenance practices that rely on frequent data collection. To address these issues, this study introduces an approach integrating supervised and reinforcement learning for predictive maintenance. The supervised learning model, developed from validated simulations, predicts wheel wear progression, while reinforcement learning enhances maintenance decision-making using operational data. This approach minimizes reliance on frequent measurements while optimizing maintenance schedules. The study fine-tunes various machine learning techniques to achieve the best performance, ensuring timely interventions to prevent wheel defects. By combining these methodologies, the proposed approach improves wear prediction accuracy ($R^2$ > 0.8), and maintenance effectiveness (most of defects can be eliminated), and mitigates unexpected failures. It offers practical solutions to industry challenges, including system complexity, limited data, and cost-effectiveness. This pioneering work demonstrates the first integration of supervised and reinforcement learning for train wheel wear management. It delivers significant benefits, including reduced maintenance costs, improved efficiency, minimized defects, shorter inspection times, enhanced passenger comfort, and increased safety, contributing to the advancement of railway asset management.

Keywords: Supervised Learning, Reinforcement Learning, Train wheel wear, Predictive Maintenance, Conditional Monitoring

## 1. Introduction

Train wheel wear occurs due to train operation and it is unavoidable. Train wheel wear is another type of wear that railway operators are concerned about besides rail wear. This study investigates the train wheel wear in this stage. Train wheel wear management is a complicated process to proceed with. Railway operators have to perform inspections and measurements before making decisions about maintenance. The inspection and measurement are time and cost-consuming which can possibly disturb regular operations. Good wheel wear management will result in good efficiency, safety, system performance, cost management, and passenger comfort [1]. For the definition, wear is the loss of material. In this case, the train wheel's mass can be lost due to the continuous employment of trains. Railway operators try to maintain the shape of the wheel in good condition due to the mentioned benefits [2].

Train wheel wear can be affected by different factors such as the weight of rolling stocks, the speed of rolling stocks, track characteristics, or types of wheels and rail material [3]. This type of wear is difficult to predict because its process is complicated [4]. Railway operators tend to generally inspect the dimensions of wheels and determine the maintenance [5] which takes time and cost. Therefore, it is usual that sometimes the inspection and measurement are insufficient [6, 7].

There are different approaches for wear maintenance such as corrective, preventive, and predictive maintenance [8]. Nowadays, predictive maintenance draws attention and tends to be continuously developed due to its advantages. However, the challenge of applying this approach is it requires data and effective tools. It has been proven that one of the effective tools is machine learning [9]. This combination can minimize unnecessary maintenance and improve system performance.

An objective of this study is to develop an approach integrating supervised and reinforcement learnings to manage train wheel wear. Data used in this study is basic operational data which is available and easy to gather. This can significantly save time and cost of inspection and measurement. Then, the reprofiling schedule will be planned based on the results from the integrated approach. The expected benefits of this study are maintenance cost reduction, maintenance efficiency improvement, defect reduction, possession and inspection time reduction, passenger comfort improvement, and safety improvement.

## 2. Literature review

Train wheel wear is one of the common issues in the railway industry. Railway operators try to maintain their system efficiency by maintaining railway components in good condition while minimizing the cost. Pascual and Marcos [10] developed the corrective maintenance approach by improving the wheel designs. They aimed to reduce flange wear and concluded that the design was related to wear management. Muhamedsalih et al. [11] applied a numerical model to forecast wheel wear and suggested that wheels should be turned every 100,000 miles. In the past, wheels were measured to determine the current dimensions. When any dimensions exceed thresholds, maintenance will be done [12]. As mentioned, the inspection and measurement are time and cost-consuming. Therefore, there were attempts to develop advanced techniques to evaluate wheel conditions such as laser [13-16], ultrasound or ultrasonic [17-20], acoustic emission [21, 22], computer vision [23-25], machine learning [4, 26-29], or combined techniques [30].

For machine learning techniques, Shebani and Iwnicki [4] applied an artificial neural network (ANN) to predict wheel wear. Their model yielded the mean absolute percentage error (MAPE) between 6.63% and 11.37%. Liu et al. [31] also applied ANN and Long short-term memory (LSTM) to predict train wheel wear. Their model also provided good performance. Chen et al. [32] applied axle box acceleration (ABA) as a feature and developed a regression model. They got a MAPE of 10% and an $R^2$ of 0.9457.

When train wheel wear is forecasted, railway operators will decide about the maintenance. The maintenance can be scheduled based on experience, historical data, or maintenance schedule. However, there was evidence that reinforcement learning (RL) yielded satisfying performance. Mohammadi and He [33] applied RL to schedule maintenance based on the Track Quality Index (TQI). Sresakoolchai and Kaewunruen [34] applied deep RL integrated with digital twins by using multiple sources of data and including different types of railway maintenance. They found that the RL model could reduce the number of defects and maintenance costs. This phenomenon was also confirmed by different studies [35, 36].

From the literature review, the integration between supervised learning and RL for train wheel wear has not been investigated. This study aims to explore the possibility of the approach by using basic operational data to manage railway wheel wear. Because there are different types of train wheel wear, this study will focus on tread wear, flange height, and flange thickness. This study will use a validated multi-body simulation (MBS) model to generate data for supervised learning to predict wear in different aspects as mentioned. The sample size is bigger than 44,000. Then, the RL model is developed to consider the maintenance schedule. For simulations, data variation is considered by varying weights of rolling stock, speed of rolling stock, and track characteristics (straight, curve, and spiral). The RL model also considers the operational uncertainties such as distances of operation, weights of rolling stock, and wear. This is to ensure that the conditions of machine learning model training are similar to the real characteristics as much as possible.

## 3. Train Wheel Wear Model and Validation

This study applies the MBS concept to generate numerical data. The software used in this study is Universal Mechanism (UM). For the rolling stock, the case study is the C80 freight car shown in Fig. 1.

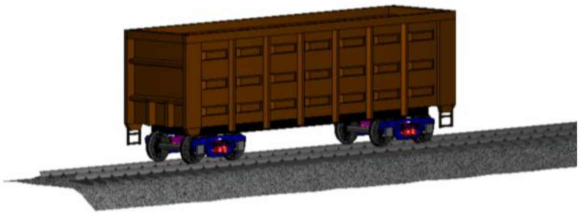


**Fig. 1** C80 freight car MBS model.

From the figure, the assembled components consist of wheels, suspension systems, bogies, car bodies, tracks, track structures, and track irregularities. For wear simulation, different wear models can be used. However, in this study, the Archard model is used because the performance is outstanding. The wear model can be considered based on Eq. (1) to Eq. (3). where $W$ is wear, $k_v$ is wear coefficient, $A$ is friction work, $P$ is the power of frictional forces, $\tau$ is tangential traction, $s$ is sliding velocity, and $f$ is contact patch area.

$$W = k_v A \tag{1}$$

$$A = \int_0^t P dt \tag{2}$$

$$P = \int_F \tau s d f \tag{3}$$

To validate the MBS model, the results from UM are compared with the calibrated data. The validation has been done in Ref. [37] and Ref. [38] comparing results from NUCARS, UM, and field data. The comparison is shown in Fig. 2.

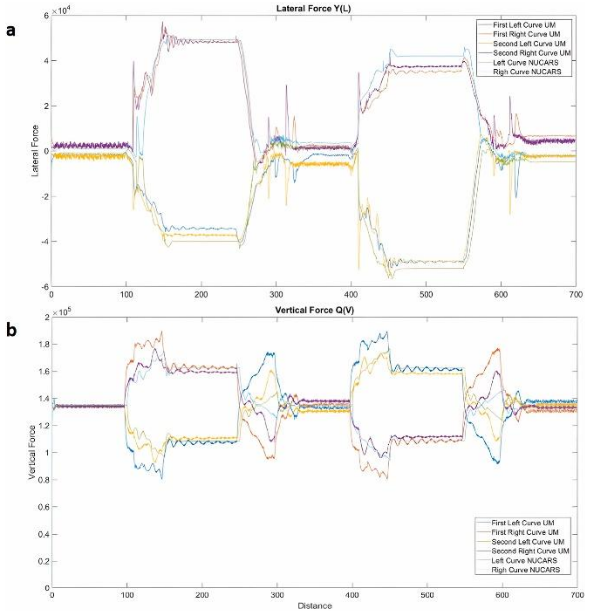



**Fig. 2** Comparison of lateral and vertical forces between the benchmark using NUCARS and UM [38].

From the figure, the differences between NUCARS and UM are small. Lateral forces and vertical forces are comparison criteria. The differences are smaller than 10% showing that UM can be used to represent the wear behavior [39-45].

## 4. Train Wheel Wear Simulation

As mentioned, data variation is included in the MBS simulations. Different characteristics of the track are considered, namely, tangent tracks and curved tracks. Other parameters are the weights of rolling stocks and other operational uncertainties. Fig. 3. presents the wheel in perfect condition and the train is operated for 999,000 km when the weight of the rolling stock is 14.5 tonnes.

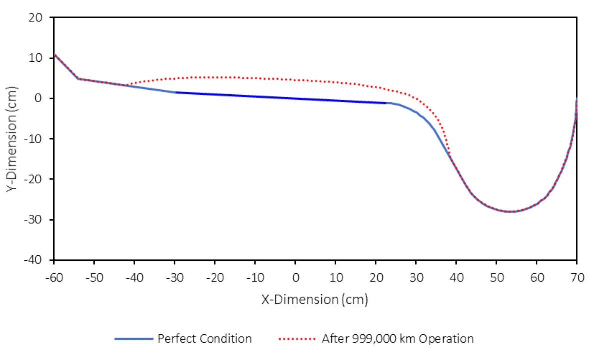



**Fig. 3** Wheel profile under different conditions.

To consider the different aspects of wear, Fig. 4 demonstrates different aspects and how to measure them.

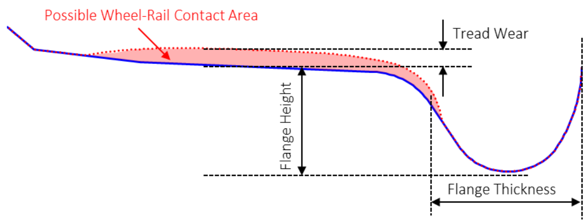



**Fig. 4** Different aspects of train wheel wear.

To develop the supervised learning model, data variation is considered as shown in Table 1. From these variations, the total number of samples for the supervised learning model is more than 44,000 samples.

**Table 1** Data variation.

| Parameters | Variation | Units |
|---|---|---|
| Track characteristic | Tangent/curve | N/A |
| Track's radii of curvature | 0 – 650 | m |
| Weight of rolling stock | 14.5 – 84.5 | Tonnes |
| Mileage | 0 – 999,000 | km |
| Operational quantity | 0 – 69 | Million Tonne-km |

## 5. Wear Prediction using Supervised Learning

Supervised learning technique is used to develop a predictive model to predict different aspects of wear consisting of tread wear, flange height, and flange thickness as mentioned. The features used to train the supervised learning model are the characteristics of tracks, the track's radius of curvature, the weight of rolling stock, mileage of operation, and operation quantity.

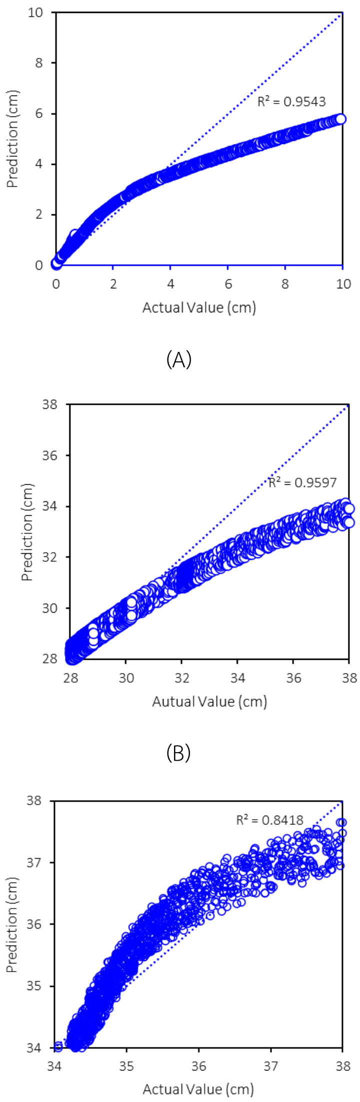
Data splitting is conducted with a proportion of 70/30 for the training and testing model respectively. In this study, a deep neural network (DNN) is used to develop the predictive model. Performance evaluation is conducted based on two indicators, Mean Absolute Error (MEA) and $R^2$. Hyperparameter tuning using grid search is done to ensure the performance of the model.

From the predictive model development, the performance of the model is shown in Table 2.

**Table 2** Model performance.

| Label | MAE (mm) | $R^2$ |
|---|---|---|
| Tread wear | 1.33 | 0.95 |
| Flange height | 1.02 | 0.96 |
| Flange thickness | 1.15 | 0.84 |

To demonstrate the model's performance in detail, Fig. 5 presents the relationships between actual values and prediction values from the model. For the perfect situation, the prediction whould be equal to the actual value. However, in reality, it is impossible to develop the prediction model to achieve $R^2$ equal to one.




(C)

**Fig. 5** Relationship between actual and prediction values from the supervised learning model; (A) tread wear, (B) flange height, and (C) flange thickness.

From the figure, the range of $R^2$ is 0.84 to 0.96 while the MAE from the table is about mm for every aspect of train wheel wear showing the good performance of the predictive model. From hyperparameter tuning, the DNN model consists of two hidden layers when the number of hidden nodes is five for both of the hidden layers. The activation functions in hidden layers are ReLU (Rectified Linear Unit) while the activation function of the output layer is linear. The dropout layer is placed prior to the output layer with a proportion of 0.2 to prevent overfitting.

## 6. Reinforcement Learning (RL) for Maintenance Schedule

A capability that makes RL different from supervised and unsupervised learning is RL can continuously make decisions based on previous decisions. To code the RL model for a particular problem, an environment is designed to fulfill this requirement. The environment in the RL refers to the rule for the agent to follow. The agent will receive information from the environment. Then, the agent has to take action by choosing an action from action spaces or available actions. Then, the environment will provide feedback to the agent telling whether the chosen action is good or bad. Information provided by the environment is called states. Timesteps that the agent has to take action are stages. Feedback provided to the agent is the reward which can be positive or negative (penalty) depending on the designed environment. The objective of the agent is to maximize rewards. The process of decision-making will be repeated until the defined last stage.

Deep Q-learning (DQN) is used to develop the RL model in this study due to its benefits, namely, flexibility and scalability. DQN is developed based on Q-learning. For Q-learning, rewards are stored in the Q-table showing the relationship between states and actions. From this concept, Q-learning will face the challenge when the environment is complicated because it is almost impossible to store rewards related to every state. As a result, the performance of the model will drop when the agent has to deal with states that it has never been trained. DQN is developed to dominate this limitation by using another DQN model to determine Q-values under different states and actions.

## 7. Problem Description

Some states for the RL model are predicted by using the developed DNN model presented previously. Features for the DNN model are pre-defined based on field data such as track characteristics, the track's radius of curvature, the weight of rolling stock, mileage of operation, and operation quantity. Then, different aspects of wear are predicted using the developed DNN model. Pre-defined features, predictions from the DNN model, and the previous action of the agent in the RL model are used as states for the RL model. In the first stage, the dimensions of the train wheel are perfect. Then, under the employment condition, the wear will emerge based on the operation. Some operational parameters are varied such as the weights of rolling stocks to mimic the real characteristics. In addition, the uncertainties of wear shown as MAE in Table 2 are also included. For the stage, it is defined based on the distance of the operation of 1,000 km. From the literature review, train wheels can be operated up to 1,120,000 km [46] because they need to be changed. Therefore, the last stage is 1,120.

In this case, the RL agent has two available actions, perform or not perform wheel reprofiling. If the agent chooses to reprofile the wheel, every dimension will return to the original value or perfect condition. If not, different aspects of wear will increase cumulatively from the previous stage. Rewards for the agent are designed consisting of two parts. The first part is based on the dimensions of the wheel, namely, tread wear, flange height, and flange thickness. The second part is based on the maintenance conducted. For the wear aspects, if any wheel's dimensions exceed the threshold, a relatively high penalty will be provided to the agent. On the other hand, the agent has to be prevented from keeping doing the maintenance. Therefore, the agent also receives the penalty when the agent chooses to perform the maintenance however the size of this penalty is relatively small compared to the defect penalty. This is to ensure that the agent will perform the maintenance as less as necessary. For the thresholds, this study refers to GMRT2466 [47], and the defined thresholds are shown in Fig. 6.

The performance of the RL model will be determined by comparing the results from the RL model and the routine maintenance. For routine maintenance, the inspection is done every 50,000 km [48-50] so the measurement will be done at a certain distance. If the wheel dimensions do not exceed the thresholds, they will be employed for another 50,000 km before being inspected again. The workflow of the RL model is shown in Fig. 7.

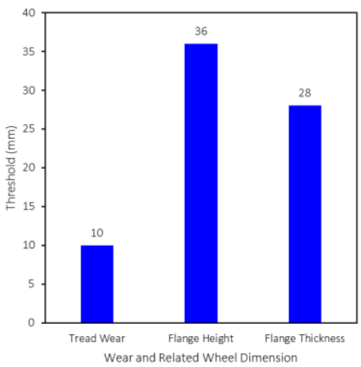



**Fig. 6** Wear thresholds.

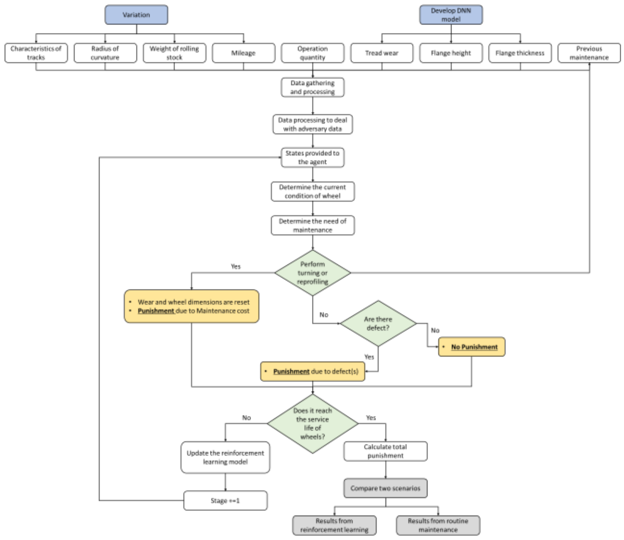



**Fig. 7** RL model workflow.

## 8. Results

In this study, the DNN model is embedded in the RL model to predict different aspects of train wheel wear as mentioned. The railway operators use their own data to train the supervised and RL models to suit their problems. Fig. 8 presents the comparison results between the RL model and routine maintenance.

The figure presents the comparison of the number of individual defects, combined defects, and performed maintenance. For the definitions, individual defects mean tread wear, flange height, and flange thickness will be considered separately. If tread wear and flange height exceed the thresholds in a stage, they are still considered separately or the number of defects is two. For the combined defects, train wheels will be considered whether they are defective. Therefore, no matter how many types of defects the wheel has in a stage, they are counted as one. That is why the number of combined defects is lower than the number of individual defects.

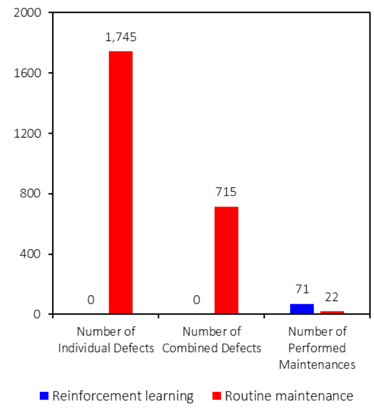


**Fig. 8** Performance comparison between the RL model and routine maintenance.

From the figure, the number of defects when applying the RL model is zero or no defect while the number of defects when routine maintenance is applied is high, this is because of the limitation of routine maintenance that the inspection and measurement are not conducted frequently enough. Therefore, the identification of defects is not responsive enough. However, the application of the RL model yields a higher number in performing maintenance which is a trade-off when the train wheel needs to be kept in good condition. To deliver a clearer view, Fig. 9 presents the number of individual defects compared to the mileage for the first 50,000 km for clearly demonstration. It is worth noting that the the mileage of the operation will extend until 1,120,000 km or the service life of wheels.

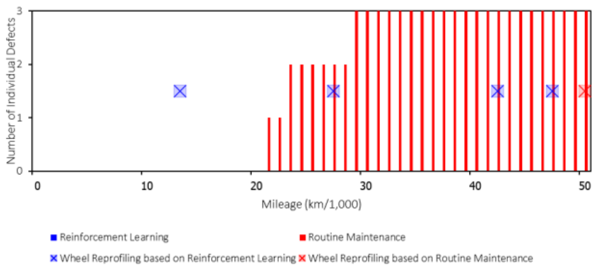



**Fig. 9** The number of individual defects and mileage.

From the figure, during the first 50,000 km of operation, the RL model can detect the requirement of maintenance or reprofiling four times and acknowledge the responsible persons for performing the maintenance. Therefore, there is no defect occurring when the RL model is used to schedule the maintenance. However, for routine maintenance, the inspection and measurement are done after the operation of 50,000 km. Therefore, the defective wheel has not been detected until that point. The defect occurs along the operation starting from flange thickness after the 21,000-km operation, tread wear after the 23,000-km operation, and flange height after the 29,000-km operation.

To present the changes in different aspects of wear, Fig. 10 is presented. From the figure, after the reprofiling is conducted, the wear and related dimensions will be decreased in the case of the RL model. For routine maintenance, the different aspects of wear will be worse along the operation because the wheel is not maintained until the 50,000-km operation.

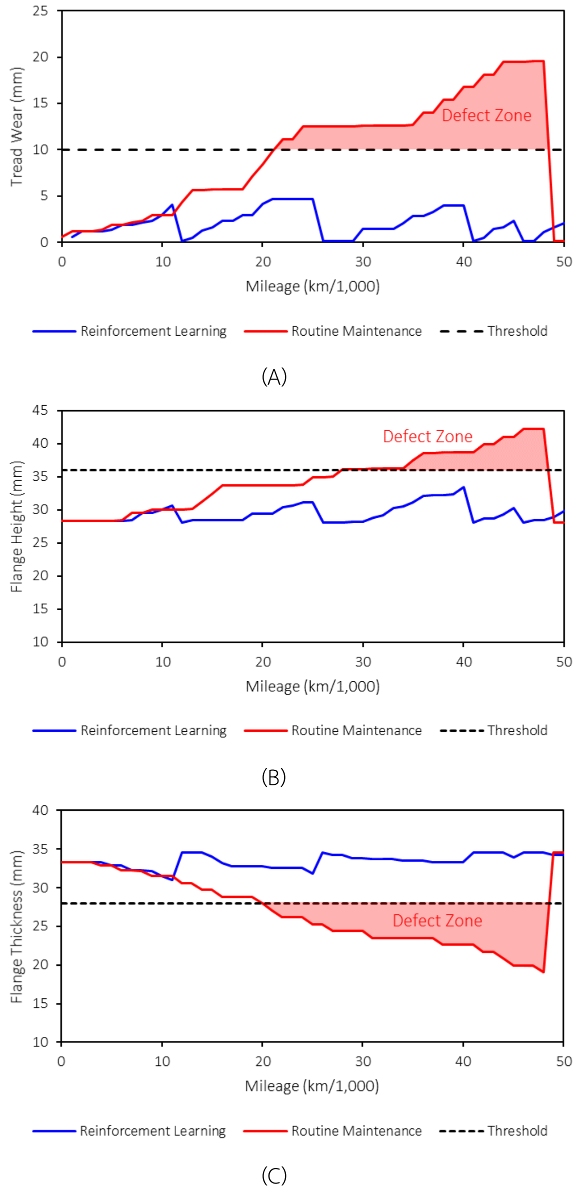



**Fig. 10** Different aspects of wear and mileage; (A) Tread Wear, (B) Flange Height, and (C) Flange Thickness.

From the results, it can be concluded that the integrated supervised and RL models can improve the maintenance efficiency of the railway system by reducing the number of emerging defects and maintaining train wheels in good condition regularly.

## 9. Conclusion

This study develops the integrated supervised and reinforcement learning approach to manage train wheel wear using basic operational data. The developed approach provides research novelties, including maintenance schedule optimization using basic operational data, cost and time savings for railway operators, and a robust framework for wheel wear management that incorporates various wear considerations, track characteristics, and operational uncertainties.

The supervised learning model effectively predicts different aspects of train wheel wear using over 44,000 samples, with DNN achieving outstanding accuracy ($R^2$ > 0.84) and an MAE of approximately 1 mm. The RL model, developed using the DQN technique, optimizes maintenance scheduling by balancing reprofiling actions against defect risks, leading to an efficient, data-driven decision-making process. Results demonstrate that reinforcement learning can significantly reduce defects while minimizing unnecessary maintenance.

By implementing this hybrid learning approach, railway operators can enhance maintenance efficiency, reduce costs, improve passenger comfort and safety, and decrease inspection and possession time. The proposed framework can be adapted to different operational constraints by incorporating railway-specific data and additional complexities, which future research can further explore.

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