Automatic Adaptive SBFE-SERA Approach for Structural Topology Optimization under Dynamic Forces

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Abstract

Structural topology optimization under dynamic loads has become increasingly important for designing efficient structures. This study investigates topology optimization techniques by adopting the Scaled Boundary Finite Element Method (SBFEM) and the Sequential Element Rejection and Admission (SERA) algorithm to optimize 2D structures subjected to dynamic loads. The SERA method creates virtual and real domains, which are sorted independently. It is demonstrated that optimization results differ when a single list of sensitivity numbers is used, leading to distinct additions or removals of elements. In dynamic response topology optimization problems that rely on time-dependent analysis, the computational effort is significantly higher due to the need for time integration. The time-history equivalent static load is computed using high-order implicit time integration with partial fractions based on Padé expansions. Additionally, to broaden the application of structural topology optimization, this study incorporates the Adaptive Design Domain (ADD) method. The ADD method significantly improves structural performance by simultaneously optimizing the material layout and expanding the design space.

Keywords: Scaled boundary finite element, Topology optimization, Sequential Element Rejection and Admission, Implicit time integration, Padé expansion

1. Introduction

Topology optimization (TO) is a powerful computational method that integrates engineering and architectural principles to determine the optimal material distribution within a predefined design domain, under specified loads and boundary conditions [1, 2]. Among the various algorithms employed in TO, evolutionary-based approaches are particularly popular. One notable example is Evolutionary Structural Optimization (ESO) [3], which enhances structural performance by iteratively removing material. This approach was later extended to allow both material addition and removal, resulting in the development of Bidirectional Evolutionary Structural Optimization (BESO) [4]. BESO improves structural designs by adding material to regions requiring increased stiffness and removing material from less critical areas. Despite its effectiveness, BESO can encounter challenges, such as the rapid removal of material leading to the loss of geometric integrity, potentially destabilizing the design domain. To address these limitations, Rozvany and Querin introduced the Sequential Element Rejection and Admission (SERA) method [5]. SERA creates independent virtual and real domains and demonstrates that optimization outcomes vary significantly when a single sensitivity list is used, leading to distinct patterns of element addition or removal.

A promising technique for solving diverse TO problems is the Scaled Boundary Finite Element Method (SBFEM), first introduced by Song and Wolf [6]. Recent advancements have enhanced SBFEM by enabling direct input of image files and generating meshes using quadtree or octree cells [7]. This image-based approach within the SBFEM framework simplifies mesh generation in both 2D and 3D spaces, effectively addressing issues related to hanging nodes. It also reduces the number of potential element realizations, resulting in faster computational performance while lowering memory and computational requirements compared to uniform mesh schemes [8].

Traditionally, TO formulations are based on linear elastic conditions under static loads, referred to as elastostatic conditions. However, real-world structures are frequently subjected to dynamic loads, necessitating analysis as elastodynamic systems. Dynamic loads, characterized by time-dependent variations, induce stresses and displacements that evolve over time. Consequently, incorporating dynamic load effects in TO is crucial to ensure structural safety, durability, and cost-efficiency.

The challenges and opportunities highlighted above form the basis of this study, which explores the application of SBFEM [9] for dynamic response topology optimization using the SERA algorithm to enhance computational efficiency. This work employs an image-based SBFEM framework with automated quadtree mesh generation, reducing manual intervention. An adaptive mesh scheme, implemented using a convolution filtering algorithm, further minimizes computational overhead. For the optimization process, the objective function is demonstrated with examples using the Equivalent Static Load (ESL) method. Displacement calculations at each time step are performed using Padé approximations via partial fractions to explore diverse solutions. Additionally, this study incorporates the Adaptive Design Domain (ADD) method, which simultaneously optimizes material distribution and expands the design space, thereby significantly enhancing structural performance. Overall, this work aims to advance the application of structural topology optimization [10].

2. Dynamic Response Topology Optimization

2.1 Problem Formulation

The compliance of the structure is evaluated in the time domain to improve structural performance, boost stability, and maintain safety. The objective function is formulated as a weighted summation of the compliance near critical peak points. This process involves assigning weights to the compliance values at specific times and minimizing the overall weighted sum of these compliances, as illustrated in the following equation:

$$\text{Minimize:} \quad \mathbf{C} = \sum_{m=1}^{\psi} \omega_m (\mathbf{f}_{t_m}^T \mathbf{u}_{t_m})$$

$$\text{Subject to:} \quad \begin{cases} \mathbf{M}(\mathbf{x})\ddot{\mathbf{u}}(t) + \mathbf{C}(\mathbf{x})\dot{\mathbf{u}}(t) + \mathbf{K}(\mathbf{x})\mathbf{u}(t) = \mathbf{f}(t) \\ V(\mathbf{x}) = \sum_{e=1}^{N_e} \mathbf{x}_e v_e - V^* = 0 \\ \mathbf{x}_e \in \{\mathbf{x}_{\min}, 1\} \end{cases} \tag{1}$$

where ψ is the total number of time steps near the peaks, ω_m is the weighting factor, v_e is the volume vector of each element, V^* is the specific total volume of the structure, N_e is the number of elements.

2.2 SERA Algorithm

The Sequential Element Rejection and Admission (SERA) algorithm operates by considering two distinct domains represented by separate material models: real material and virtual material. The separation of these domains is guided by sensitivity analysis, which determines whether elements should be added or removed to achieve the desired volume fraction. The final optimized topology is composed solely of real material, achieved when the objective function is minimized, and the target volume fraction is satisfied. The SERA method follows a structured sequence of steps:

STEP 1: Specify the maximum design domain and discretize it for finite element analysis (FEA).

STEP 2: Calculate the variation in volume fraction, including the quantities to be added and removed during each iteration.

STEP 3: Compute the sensitivity of each element, which reflects its contribution to the structural performance.

STEP 4: Implement a filtering process to the sensitivity values to enhance accuracy and mitigate checkerboarding effects.

STEP 5: Use the SERA update function to modify both real and virtual material domains based on sensitivity analysis results.

STEP 6: Assess convergence by evaluating the stopping criterion to determine if the optimization process has achieved satisfactory results.

STEP 7: Repeat the procedure until the target volume fraction is reached and the design converges to an optimal solution.

2.3 ESLSO for Dynamic Response

The ESL method for nonlinear static response structural optimization (ESLSO) involves converting nonlinear static response problems into a series of equivalent linear static load cases. This transformation simplifies the optimization process, allowing traditional static optimization techniques to be applied. Loads that produce the same displacement field as dynamic loads at each time step are referred to as ESLs. The governing equation for dynamic response analysis can be modified by defining the vector of ESLs as:

$$\mathbf{f}_{eq}(t) = \mathbf{K}(\mathbf{x})\mathbf{u}(t) = f(t) - \mathbf{M}(\mathbf{x})\ddot{\mathbf{u}}(t) - \mathbf{C}(\mathbf{x})\dot{\mathbf{u}}(t) \tag{2}$$

$$\mathbf{f}_{eq}(s) = \mathbf{K}(\mathbf{x})\mathbf{u}(s) \tag{3}$$

where $\mathbf{u}(s)$ is the dynamic displacement at each interval in time

2.4 Sensitivity Analysis

Direct optimization based on time-domain motion equations typically requires solving these equations at every time step, followed by sensitivity analysis, which can significantly increase computational demands. To mitigate this challenge, the proposed method computes elemental sensitivity numbers by focusing on critical points, such as peaks and their surrounding regions, identified through time-dependent analysis. This approach is expressed as follows:

$$\alpha_e = -\frac{1}{p}\sum_{m=1}^{\psi}\frac{\partial \mathbf{C}}{\partial \mathbf{x}_e} = \mathbf{x}_e^{p-1}\sum_{m=1}^{\psi}\left(\frac{1}{2}\mathbf{u}_e^T\mathbf{K}_e\mathbf{u}_e\right) \tag{4}$$

The sensitivity number assigned to each element plays a pivotal role in determining whether elements should be added or removed during the optimization process. The primary objective of the sensitivity analysis is to minimize the compliance of the structure, thereby improving its stiffness and overall performance under applied loads [11].

Elements with lower sensitivity numbers are typically prioritized for removal or addition, as these modifications yield the most significant impact on reducing the structure's compliance. Interestingly, adding elements with lower sensitivity values can be interpreted as functionally equivalent to adding elements with higher sensitivity values, provided the sign of the corresponding sensitivity number is inverted.

This dual perspective introduces greater flexibility and adaptability into the optimization process, allowing the algorithm to strategically refine material distribution. As a result, the optimization process achieves enhanced structural efficiency by balancing material allocation to maximize performance while minimizing compliance.

2.5 Filtering of Sensitivities

The sensitivity numbers are first smoothed by applying a weighted average of all individual compliances to avoid checkerboard patterns and mesh dependency, as follows:

$$\alpha_e = \frac{\sum_{j=1}^{\Gamma}\Delta(r_{ej})\alpha_e}{\sum_{j=1}^{\Gamma}\Delta(r_{ej})} \tag{5}$$

$$\Delta(r_{ej}) = \max(r_{\min} - r_{ej}, 0) \tag{6}$$

where α_e is the filtering sensitivity value of individual elements, $r_{\min}$ is the filtering radius, Γ is the set of all element with a straight-line distance from the center point of the element within the filtering radius, r_{ej} is the distance between the center of elements e and j.

2.6 Optimal Criterion

In each iteration of the design process, the target volume for the current iteration is set in advance. The initial design domain begins with a full solid mass, so the target material volume must be less than the initial volume. This procedure is repeated until the desired volume is attained, and the objective function values must meet the criteria for stopping the evolutionary process:

$$C_{change} = \frac{\left|\sum_{i=1}^{N}C_{\kappa-i+1} - \sum_{i=1}^{N}C_{\kappa-N-i+1}\right|}{\sum_{i=1}^{N}C_{\kappa-i+1}} \le C_{tol} \tag{7}$$

where C_{tol} is the defined convergence tolerance, N is an integral number.

3. Numerical Examples

3.1 Cantilever Beam with a Half-Cycle Sinusoidal Load

The first example in the proposed design optimization system involves a cantilever beam subjected to dynamic loading following a half-cycle sinusoidal pattern. This study examines three load cases, defined by time increments t_f of 0.1 s , 0.01 s, and 0.001 s, comparing the performance of the Sequential Element Rejection and Admission (SERA) method with the Bidirectional Evolutionary Structural Optimization (BESO) method. The cantilever beam has a width of 512 pixels and a height of 256 pixels. The left edge is fixed, while the right edge remains free, as shown in Figure 1. The material properties are initialized with a Young's modulus of 210 GPa and a density of 7.8 kN/m^3.

The dynamic loading (f) is applied at the center of the free end on the right side, as illustrated in Figure 1. The load follows a half-cycle sinusoidal pattern, with three simulation time cases used for comparison. The damping ratio is set to zero to simplify the analysis.

Fig. 1 Cantilever beam with a half - cycle sinusoidal load.

The optimization parameters such as The desired remaining volume fraction is 0.5 ,PR = 5 % , SR = 1.3, and B = 0.003. The element size is restricted to ≤ 32 pixels and ≥ 2 pixels.

The results of the cantilever beam problem under a half-cycle sinusoidal load are presented in Table 1, comparing the performance of the SERA and BESO approaches. Both methods achieved similar compliance values in static design. However, in the dynamic case with a simulation time increment, the Equivalent Static Load Sequential Optimization (ESLSO) design exhibited higher compliance compared to the SERA approach, indicating a less efficient material distribution.

Table 1 Results of cantilever beam with a half-cycle sinusoidal load.

Algorithm	t_f (s)	Iterations	Avg. Time (s)	Compliance From ESLSO design	Optimal result
SERA	0.1	122	10.51	0.000438	
	0.01	122	3.47	0.000438	
	0.001	122	2.50	0.000410	
BESO	0.1	40	11.96	0.000872	
	0.01	34	3.08	0.000437	
	0.001	28	2.38	0.000411	

In terms of computational efficiency, the BESO method required fewer iterations than the SERA approach, resulting in a shorter total computation time. This advantage highlights BESO's computational efficiency, particularly for scenarios requiring rapid optimization.

Regarding volume fraction behavior, the SERA approach showed a gradual decrease in volume fraction over the course of optimization, enabling a more controlled design evolution. In contrast, the BESO approach exhibited a more rapid reduction in volume fraction, which may lead to less gradual transitions in the material distribution.

3.2 Cantilever Beam Under Two Dynamic Loads

In this example, a rectangular cantilever beam is subjected to two dynamic loads applied simultaneously, as shown in Figure 2. To facilitate a comparison between the SERA and BESO methods, the results are analyzed under the same simulation time of 0.2 s, with varying maximum element sizes of 8 pixels, 16 pixels, and 32 pixels. The material properties are initialized with a Young's modulus of 210 GPa and a density of 7.8 kN/m^3, likewise the previous example.

In this example, two dynamic loads, with force $\mathbf{F}_1(t) = 1000\sin(\pi t/t_f)$ kN applied at the top of the free end on the right side in the downward direction, and force $\mathbf{F}_2(t) = 2000\cos(\pi t/t_f)$ kN applied at the bottom of the beam's midpoint in the upward direction. The simulation time is 0.2 s. The optimization parameters such as the desired remaining volume fraction is 0.5, PR = 5 %, SR = 1.3, and B = 0.003.

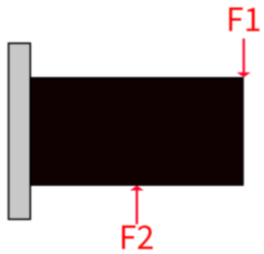


Fig. 2 Cantilever beam under two dynamic loads.

The results for the cantilever beam subjected to two dynamic loads are presented for three cases, each corresponding to different maximum element sizes: 8 pixels, 16 pixels, and 32 pixels. The performance of the BESO and SERA approaches was compared in terms of compliance and computational efficiency, are presented in Table 2.

Table 2 Results of the cantilever beam under two dynamic loads.

Algorithm	Max. Size (px.)	Iterations	Avg. Time (s)	Compliance From ESLSO design	Optimal result
SERA	8	41	158.39	0.060765	
	16	42	160.46	0.059306	
	32	40	134.39	0.059421	
BESO	8	39	133.97	0.060954	
	16	29	125.96	0.059410	
	32	36	148.13	0.060955	

As shown in Figure 3, the compliance of the ESLSO design obtained using the BESO approach was higher than that achieved with the SERA approach across all cases. Notably, in the case with a maximum element size of 8 pixels, significant jumps in compliance values were observed, indicating potential sensitivity to mesh refinement in this scenario.

In terms of computational time, both the BESO and SERA approaches yielded comparable results, demonstrating similar levels of efficiency despite their distinct optimization strategies

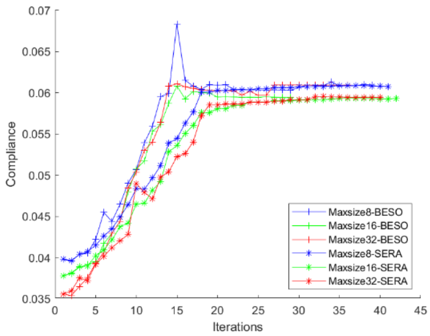


Fig. 3 Evolution histories of compliance for the BESO and SERA methods applied to a cantilever beam under two dynamic loads.

3.3 Adaptive Design Domain of a Cantilever Beam Under Two Dynamic Loads

In this example, a cantilever beam is subjected to two dynamic loads within an adaptive design domain. The design domain is extended to include two additional layers, each measuring 32 pixels in size. The input image for the domain has dimensions of 512 pixels × 1024 pixels, with the cantilever beam represented as a black rectangle of 256 pixels × 512 pixels.

The material properties are initialized with a Young's modulus of 210 GPa and a density of 7.8 kN/m^3. The two dynamic loads are applied as follows: the first load is positioned at the top right corner of the beam, while the second is applied to the bottom middle, as shown in Figure 4

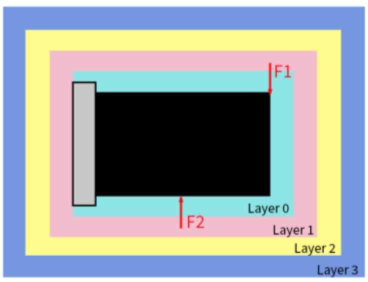
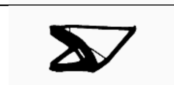


Fig. 4 ADD of a cantilever beam under two dynamic loads.

Prior to setting support conditions and loads, the black area representing the beam must be identified to define the nodes accurately. The simulation is conducted with a total time of 0.2 s, and the optimization parameters are as follows: the desired remaining volume fraction is 0.5, PR = 5 %, SR =1.3, and B = 0.003.

The results for the cantilever beam subjected to two dynamic loads within an adaptive design domain are presented, with the design domain expanded from layer 0 to layer 3. Each layer has a thickness of 32 pixels, and the optimization process utilizes the SERA approach. Key results, including iterations, compliance from the ESLSO, and volume fractions, are summarized in Table 3. The evolution histories of compliance and volume fraction are depicted in Figure 5.

Table 3 Results for a cantilever beam subjected to two dynamic loads within an adaptive design domain.

Layer	Iterations	Compliance From ESLSO design	Optimal result
0	37	0.06671	
1	74	0.057116	
2	111	0.056121	
3	148	0.055747	

In layer 0, the compliance initially increased to a peak value before stabilizing at a constant level, requiring 37 iterations to achieve convergence for each layer. Upon expanding the design domain to include additional layers, the compliance progressively decreased, demonstrating the benefit of an adaptive design space in improving structural performance.

Regarding volume fraction, layer 0 showed a noticeable reduction during optimization. However, after expanding the design domain, the volume fraction remained constant, indicating that the material layout stabilized as the optimization process progressed.

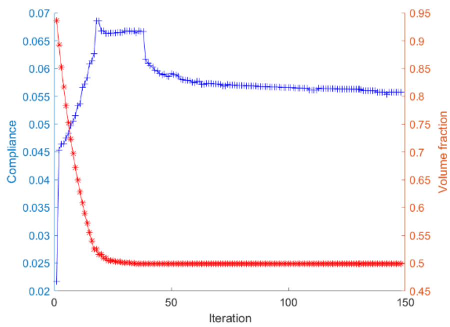


Fig. 5 Evolution histories of compliance and volume fraction for a cantilever beam subjected to two dynamic loads within an adaptive design domain.

4. Conclusion

This paper presents the integration of the advanced numerical technique, the Scaled Boundary Finite Element Method (SBFEM), with the novel Sequential Element Rejection and Admission (SERA) optimization algorithm to address topology optimization problems in structures subjected to dynamic loading. Recognizing the computational demands of time-dependent analysis, this work employs a weighted summation of critical peak points instead of full time-domain integration, significantly reducing computational effort without compromising accuracy. To further enhance solution accuracy, high-order implicit time integration, specifically partial fraction methods based on Padé expansions, is utilized for elastodynamic systems, ensuring precise response predictions. Additionally, the Adaptive Design Domain (ADD) method is incorporated to overcome the limitations of initial boundary constraints. This approach enables the design space to evolve dynamically, resulting in improved mechanical stiffness and overall structural performance.

Acknowledgement

The authors sincerely thank Professor Chongmin Song for his invaluable support, guidance, and encouragement throughout the course of this work.

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