Influence of Forward Layered Models for Backcalculation Analysis Based on Falling Weight Deflectometer

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Abstract

Backcalculation analysis is a commonly used non-destructive testing (NDT) procedure for assessing the in situ material properties of pavements. The accuracy of pavement moduli estimates obtained from FWD backcalculation analysis depends significantly on the forward model employed in the backcalculation process. In order to address this issue, this paper investigates the influence of the forward dynamic model on falling weight deflectometer backcalculation analysis. A dynamic layered model is considered and a corresponding computer code is developed to assess soil system response based on recorded FWD load impulses reported in the literature. This model is applicable to flexible layered pavements. The investigation presented in this study indicates the need for a careful interpretation of the predicted backcalculation moduli which is essential for the development of dynamic backcalculation programs and application in pavement design and construction.

Keywords: backcalculation, forward dynamic model, layered system, falling weight deflectometer, non-destructive test

1. Introduction

Highway and traffic engineering are dealing with road pavement design, material used, and the operation and maintenance of the in-service roads. Following construction, it is important to verify that the subgrade subbase's characteristics were maintained at the intended values. The Falling Weight Deflectometer (FWD) test is frequently used for the nondestructive assessment of in-situ soil qualities and the structural health of in-service pavements [1-3]. More specifically, the structural conditions of in-service pavements can be assessed by FWD and the routine maintenance schedule and rehabilitation activities can be planned.

The FWD backcalculation process for predicting the elastic properties of the subgrade soils/pavement basically consists of three main components, i.e., FWD field data, the forward soil model and the optimization process [3]. FWD backcalculation procedure estimates pavement properties by considering the measured (FWD field data) and calculated deflection profiles (from forward soil model) using optimization. Most of the commercial backcalculation programs are based on static backcalculation. The static backcalculation can, however, produce the erroneous estimation of the soil elastic due to the fact that the dynamic effect is neglected in the backcalculation process. Despite the advantages in accounting for the time-dependent load and responses, the accuracy of evaluating elastic moduli based on dynamic backcalculation analyses is, however, significantly depended on the forward dynamic soil models employed in the backcalculation process [4-7]. Hadidi and Gucunski [6] investigated the reliability of different static and dynamic backcalculation procedures using probabilistic approach. Han et al. [7] developed a numerical dynamic model based on the spectral element method for the transversely isotropic layered pavement structure.

The influence of the forward dynamic model on falling weight deflectometer backcalculation analysis is investigated in this paper. The forward model is developed by using a multi layered soil [8-9]. This model is applicable to flexible layered pavements, which are composed of wearing course, base, and sub-base layers. A computer code is developed for the FWD backcalculation based on forward dynamic model. Comparison of backcalculation based on different forward soil models are made to examine their advantage and limitation in predicting the soil layer moduli values.

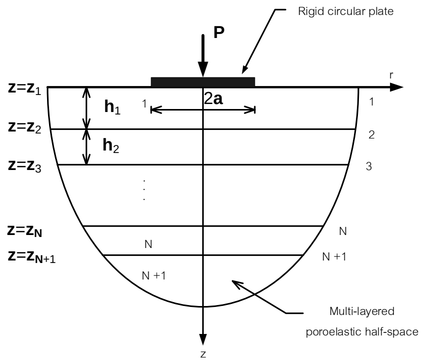

Fig. 1 Dynamic forward model considered in the present study

2. Forward Dynamic Model

This research is concerned with the study for the influence of dynamic soil models on backcalculation of engineering soil properties from the Falling Weight Deflectometer (FWD) deflection data. The forward dynamic model is formulated by using an elastic circular plate resting on a multi layered poroelastic soil [9-10]. The schematic representation for the dynamic forward model is presented in Fig. 1. The plate is subjected to axisymmetric time dependent loading and its response is governed by the classical thin-plate theory. The poroelastic soil material is governed by Biot's poroelastodynamic theory and considered as a multi-layered half-space. The constitutive relations for a homogeneous poroelastic material can be expressed as [11]:

$$\sigma_{rr} = 2\mu\frac{\partial u_r}{\partial r} + \lambda e - \alpha p\,; \quad \sigma_{zz} = 2\mu\frac{\partial u_z}{\partial z} + \lambda e - \alpha p \tag{1a}$$

$$\sigma_{zr} = \mu\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right); \qquad p = -\alpha M e + M\zeta \tag{1b}$$

where

$$e = \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} + \frac{u_r}{r}\,; \qquad \zeta = -\left(\frac{\partial w_r}{\partial r} + \frac{\partial w_z}{\partial z} + \frac{w_r}{r}\right) \tag{1c}$$

In the above equations, σ_{rr} , σ_{zz} and σ_{zr} denote the total stress component of the bulk material; u_i and w_i are the average displacement of the solid matrix and the fluid displacement relative to the displacement of the solid matrix, in the $i-$direction ($i = r, z$), respectively; p is the excess pore fluid pressure (suction is considered negative); ζ is the variation of fluid content per unit reference volume; e is the dilatation of the solid matrix; μ is the shear modulus and λ is a constant of the bulk material, respectively. In addition, α and M are Biot's parameters accounting for compressibility of the two-phased material [11].

The equations of motion for a poroelastic medium undergoing axisymmetric deformations, in the absence of body forces (solid and fluid) and a fluid source, can be expressed according to Biot [12] as

$$\mu\nabla^2 u_r + \left(\lambda + \alpha^2 M + \mu\right)\frac{\partial e}{\partial r} - \mu\frac{u_r}{r^2} - \alpha M\frac{\partial \zeta}{\partial r} = \rho\,\ddot{u}_r + \rho_f\ddot{w}_r \tag{2a}$$

$$\mu\nabla^2 u_z + \left(\lambda + \alpha^2 M + \mu\right)\frac{\partial e}{\partial z} - \alpha M\frac{\partial \zeta}{\partial z} = \rho\,\ddot{u}_z + \rho_f\ddot{w}_z \tag{2b}$$

$$\alpha M\frac{\partial e}{\partial r} - M\frac{\partial \zeta}{\partial r} = \rho_f\ddot{u}_r + m\ddot{w}_r + b\dot{w}_r \tag{2c}$$

$$\alpha M\frac{\partial e}{\partial z} - M\frac{\partial \zeta}{\partial z} = \rho_f\ddot{u}_z + m\ddot{w}_z + b\dot{w}_z \tag{2d}$$

In the above equations, $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$; ρ and ρ_f are the mass densities of the bulk material and the pore fluid respectively; and $m = \rho_f / \beta$ (β = porosity), is a density-like parameter. In addition, b is a parameter accounting for the internal friction due to the relative motion between the solid matrix and the pore fluid. The parameter b is defined as the ratio between the fluid viscosity and the intrinsic permeability of the porous medium.

Introducing the Fourier-Hankel integral transform of a temporal field $\mathbf{g}(r,t)$ via [13]

$$G(\xi,\omega) = \int_{-\infty}^{\infty}\int_{0}^{\infty} g(r,t)\, r J_n(\xi r) e^{-i\omega t}\, dr dt \tag{3a}$$

and the inverse Fourier-Hankel integral transform can be expressed as

$$g(r,t) = \int_{-\infty}^{\infty}\int G(\xi,\omega)\,\xi J_n(\xi r) e^{i\omega t}\, d\xi d\omega \tag{3b}$$

The governing partial differential equations, Eqs. (1) and (2), can be solved by applying the Fourier-Hankel integral transform given in Eq. (3). Considering a multi-layered system as shown in Fig. 1, the general solutions for solid and fluid displacements, pore pressure and stresses in the Fourier-Hankel transform space can be expressed as [9]

$$\mathrm{u}(\xi,z,\omega) = \mathrm{R}(\xi,z,\omega)\mathrm{C}(\xi,\omega) \tag{4a}$$
$$\mathrm{f}(\xi,z,\omega) = \mathrm{S}(\xi,z,\omega)\mathrm{C}(\xi,\omega) \tag{4b}$$

where

$$\mathrm{u}(\xi,z,\omega) = \begin{bmatrix}\bar{u}_r & \bar{u}_z & \bar{p}\end{bmatrix}^T \tag{4c}$$
$$\mathrm{f}(\xi,z,\omega) = \begin{bmatrix}\bar{\sigma}_{zr} & \bar{\sigma}_{zz} & \bar{w}_z\end{bmatrix}^T \tag{4d}$$
$$\mathrm{C}(\xi,\omega) = \begin{bmatrix}A & B & C & D & E & F\end{bmatrix}^T \tag{4e}$$

The matrices $\mathrm{R}(\xi,z,\omega)$ and $\mathrm{S}(\xi,z,\omega)$ in Eqs. (4a) and (4b) are given elsewhere [9]. For the nth layer (n =1, 2, 3, ..., N),

$$U_n = \begin{bmatrix} R_n(\xi, z_n, \omega) \\ \cdots\cdots\cdots \\ R_n(\xi, z_{n+1}, \omega) \end{bmatrix} C_n \tag{5a}$$
$$F_n = \begin{bmatrix} -S_n(\xi, z_n, \omega) \\ \cdots\cdots\cdots \\ S_n(\xi, z_{n+1}, \omega) \end{bmatrix} C_n \tag{5b}$$

Equation (5a) can be used to express C_n in terms of U_n and then substituted in Eq. (5b). This results in the following matrix equation.

$$F_n = K_n U_n, \ (n = 1, 2, 3, ..., N) \tag{6}$$

In the above equations, K_n can be considered as an exact stiffness matrix in the Hankel-Fourier transform space that describes the relationship between the generalized displacement vector U_n and the force vector F_n for the nth layer. The explicit derivation of K_n involves the manipulation of fully populated 6×6 complex matrices. The details are given elsewhere [9]. The global stiffness matrix of a multi-layered half-space is gathered by using the layer and half-space stiffness matrices together with the continuity conditions of fluid flow and tractions at the layer interfaces results in the following global equation system.

$$\begin{bmatrix} K_1 & & & & \\ & K_2 & & & \\ & & \ddots & & \\ & & & K_N & \\ & & & & K_{N+1} \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ \vdots \\ U_N \\ U_{N+1} \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_N \\ F_{N+1} \end{Bmatrix} \tag{7}$$

3. FWD Backcalculation

Backcalculation is an inverse problem with the objective of predicting the modulus of pavement or soil layers. The FWD backcalculation process basically consists of three main components, i.e., FWD field data, the forward soil model and the optimization process, as schematically presented in Fig. 2. FWD backcalculation procedure estimates pavement properties by matching measured (FWD field data) and calculated pavement surface deflection basins (from forward soil model) using optimization. The backcalculation elasticity modulus is determined by assuming a set of pavement layer moduli (seed moduli) in the forward soil models that can produce a deflection basin similar to those measured from the field test.

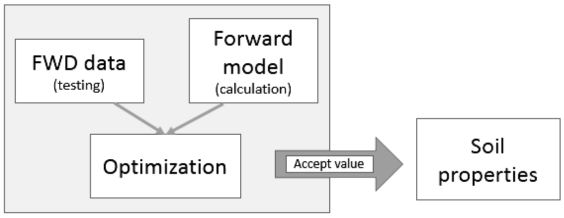

Fig. 2 Main components of FWD backcalculation process

4. Selected results and discussion

Numerical study of backcalculation based on different dynamic forward models is presented in this section. Computer programs have been developed for evaluating the backcalculation elastic modulus from the applied load pulse and the recorded deflection based on different dynamic soil models. Verification of the program was first performed by comparing with the existing studies in the literature to verify the accuracy and numerical stability of the developed dynamic soil models. The verification results are not presented in this report for brevity.

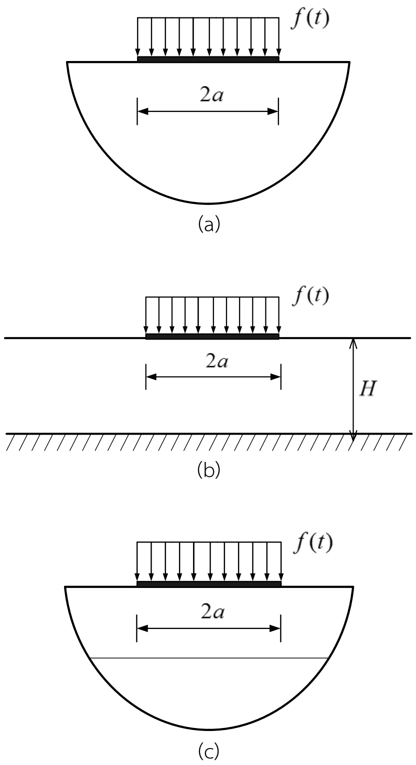

Fig. 3 Schematics of (a) half-space model, (b) rigid-base model and (c) multi-layered model

In this study, three types of soil models, namely, half-space model, rigid-base model and multi-layer model are considered as dynamic forward models in the FWD backcalculation process. The rigid-base model represents a soil system with the presence of a stiff layer. The geometry of a half-space, rigid-base and multi-layer models are presented in Figs. 3a, 3b and 3c respectively.

The applied load pulse and FWD displacement profiles for concrete crushed aggregates (CCA) soil system reported in the literature are served as the field data in the FWD backcalculation process [14]. The cross section of concrete crushed aggregates is presented in Fig. 4. The test bed are constructed by excavating the in-situ marly clay soil to a depth of 35 cm and then filled with crushed concrete aggregates (CCA). The FWD load pulse $f(t)$ by Asli et al. [14] is reproduced in Fig. 5.

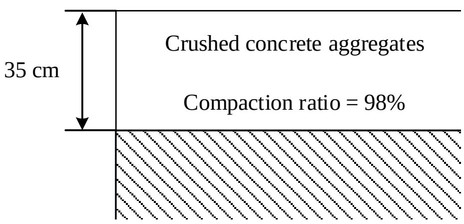

Fig. 4 Cross section of a concrete crushed aggregates soil system

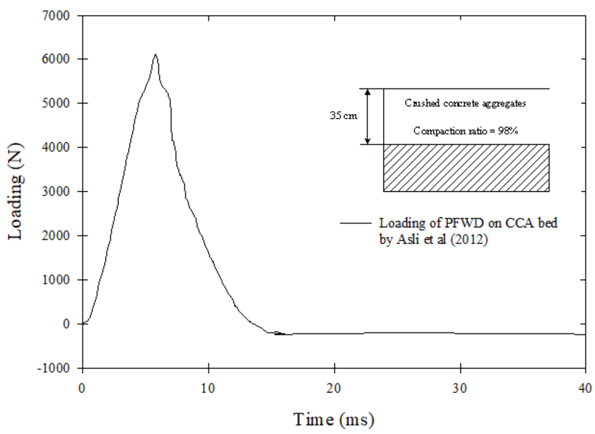

Fig. 5 Field FWD loading by Asli et al. [14]

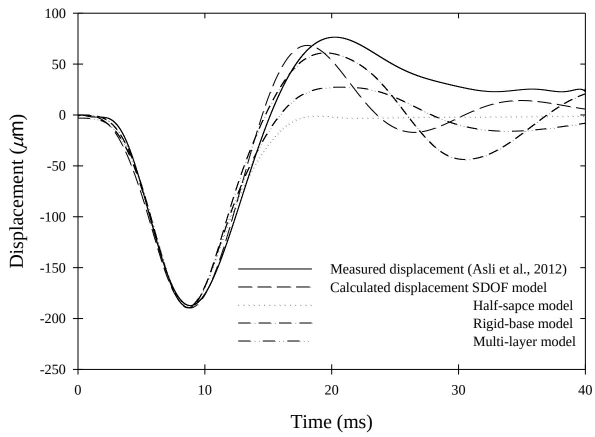

Fig. 6 Comparison of deflection profiles for concrete crushed aggregates (CCA) soil system from FWD field measurement and dynamic forward models

Fig. 6 presents a comparison between deflection profiles recorded during the FWD tests for CCA soil system [14] and those calculated using the dynamic half-space model, rigid-base model and layered model respectively. In addition, the lumped single degree of freedom (SDOF) model is considered in this study for comparison.

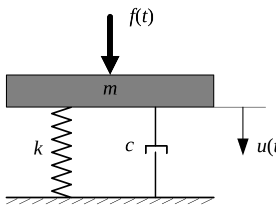

Fig. 7 Dynamic single degree of freedom (SDOF) model

The loading plate-soil system is represented by a mass-spring-damper system as shown in Fig.7, in which the dynamic governing equilibrium equation can be given by

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) = f(t) \quad (8)$$

where k, c and m are the elastic stiffness, the damping coefficient and the equivalent mass respectively; $u(t)$, $\dot{u}(t)$ and $\ddot{u}(t)$ are the deflection, the velocity and the acceleration respectively; $f(t)$ is the impulse load, and the overdot denotes the derivative with respect to the time parameter t.

The differences between the deflection profiles obtained from different forward models are clearly seen from the comparison in Fig. 6. The deflection profile obtained from the half-space model shows less oscillation compared to the measured field deflection and other dynamic forward models. This indicates that the layered model should be employed for this CCA soil system. It should be observed that although the calculated deflection obtained from SDOF and rigid-base models have bounce values (positive displacement at about the 20 ms) close to the bounce deflection measured from the field, the deflection profiles of both rigid-base and SDOF models exhibit a significant higher oscillations than the field value and with a higher frequency for the case of SDOF deflection.

It might be concluded from Fig. 6 that the dynamic multi-layer model, in general, generates a deflection profile that resembles the field FWD data. Different optimization techniques can be employed in the backcalculation procedure to predict the elastic modulus. The peak value method has been routinely used for the estimation of the soil elastic modulus and, therefore, the estimated soil modulus from backcalculation process based on different forward models are consequently different. The comparison presented in Fig. 6 suggests that the estimated soil moduli predicted from backcalculation process requires in-depth understanding of forward soil model and careful interpretation.

5. Conclusions

The influence of forward dynamic soil models on the backcalculation analysis for soil properties based on Falling Weight Deflectometer (FWD) test is investigated in this research. In the present paper, three types of dynamic forward models are considered, namely, half-space, rigid-base and multi-layer models. Extensive studies on the characteristics and performance of the proposed forward dynamic model should be further explored. In addition, the lumped single degree of freedom (SDOF) model is considered in this study for comparison. Numerical results presented in this paper indicate that the predicted elastic moduli obtained from backcalculation process are significantly depended on the forward dynamic soil models employed in the dynamic backcalculation process. The differences between the deflection profiles obtained from different forward models are clearly shown from the selected numerical example. The investigation presented in this study indicates the need for a careful interpretation of the predicted backcalculation moduli and a detailed study of the influence of the forward soil model on the backcalculation.

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