A Study of the Reinforcement of Steel Plates into RC Cantilever Stairs to Reduce Vibration Effects from Human Walking Activities

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Abstract

Static loadings resistance is always the first design criteria. However, further investigation should also be conducted for a structure like floors, bridges, and stairs which are subjected to dynamic loadings in the form of human activities such as walking, running, or jumping; these loadings often result in vibrational effects associated with the user's comfort. While most researches were mostly about walking force on flat surfaces, this study will mainly focus on reinforced concrete cantilever staircase which has been reported to encounter a significant level of vibration due to periodic walking motion. Therefore, this paper presents the attempt to study the characteristics and response of RC cantilever stair design once being subjected to human-induced vibration and to propose the application of extra reinforcing steel used to lessen the dynamic effects. Numerical analysis is done on an RC cantilever stairs model having dimensions of 100 centimeters in length, and 25 centimeters in width. To keep the concept of "modern" structures, the thickness is maintained to be only 10 centimeters. Both ascending and descending motions are performed, and acceleration data is collected for evaluation. The analysis is mainly done based on the equation of motion; thus, it is important to determine all the associated parameters such as single lumped mass, static force, damping force, and the human walking force. To obtain all these parameters, several theories are used such as force-displacement relation, composite materials (FEM), virtual work method, and Fourier series. A little experiment has to be conducted before we can use the Fourier series to get the equation that represents human walking force with respect to time. With the help of Matlab software, the equation of motion which is in the function of second order differential equation could be easily solved. For the proposed reinforcement measures to be proven effective, several parameters must satisfy the recommended design criteria which are shear force, bending moment, deflection, and acceleration. The benefits of implementing this project are reducing user's discomfort due to vibration within the staircase structure by introducing innovative reinforcement patterns while still maintaining slender dimensions as specified in the architectural plans.

Keywords**:** RC cantilever stair, human walking load, vibration on stair, human comfort vibration

1. Introduction

In civil engineering works, structures are often designed to withstand static loadings; that is uniformly distributed force. This type of force can result in deformation which every engineer must consider during the designing stage to finalize the dimensions and shapes. However, it is important to take dynamic behavior into account as well because it also imposes some noticeable deflection once being subjected to vibration due to human activities such as walking, running, jumping or intentional swaying. Not only that dynamic loadings can create significant deformation of the structures just like how static loadings do, but they are also associated with the comfort of the users and therefore fall under the serviceability limit state. This indicates that to perform design base on the static loads is not enough anymore regarding vibration issues.

Over the last decade, researchers have been very interested in studying the relationship between human-induced vibration and the response of structures toward this dynamic phenomenon. Bijan, O., and Aalami. (2010) researched design of concrete floor systems and its vibrational responses on his 8-

inch thick. The concept of modern lightweight structures has led Mohammed, AS., and Pavic, A. (2017) to conduct a study regarding the effects of walking people on dynamic properties of floors. Gaile, L. and Radinsh, I. (2013) saw that only a few works were done on slender stairs, so they decided to conduct some experiments to see their characteristics and responses. They discovered that there were excessive vibrations within the structures; that is why Pedro Andrade and Jose Santos proposed a case study about the most effective reinforcement measures in September 2017. Another relevant work was recently done by two Burapha University students in 2018; it was about the dynamic response of concrete stairs with various dimensions, but only the one with a thickness of 16 centimeters proved to be effective in reducing the dynamic effects. Because most of the previous researches are done focusing on human walking forces on a flat surface, there is little work done on stairs. The reinforced concrete structures prove to be safe, but the vibrational acceleration is just too much that humans can still perceive it. In short, the lack of comfort provided by the existing stairs design is the main target which is needed to be studied in this project. The specific type of stairs that is chosen to be focused on is cantilever stairs because they are usually the one that poses the greatest problem. This paper presents a new method of reducing dynamic response where extra reinforcement of steel plates are installed into RC cantilever stairs with a stepped thickness of only 10 centimeters. It can be done by utilizing two objectives which are: (1) To study the characteristics and response of RC cantilever stair design once being subjected to dynamic loading which is human-induced vibration and (2) To investigate and propose the application of extra reinforcing steel plates which are used to lessen the effects of dynamic loading developed within the structure.

2. Background

Before proceeding with any further analysis, it is very important to go through some brief background of the research in order to get a clearer understanding of the whole study. Regarding all factors, RC cantilever stair system can be best represented by a simple spring system. The equation governing the displacement can be derived from Newton's second law.

$$\Sigma F = ma \tag{1.1}$$

$$-f_s(t) - f_D(t) + p(t) = ma(t) \tag{1.2}$$

$$ma + f_D(t) + f_s(t) = p(t) \tag{1.3}$$

$$m\ddot{u}(t) + c\dot{u} + ku(t) = p(t) \tag{1.4}$$

Where, m is the mass of cantilever stair under single lump mass system, a is acceleration of structure after applying walking load force, f_s is the static force, f_D is damping force, p is force from human walking motion, u is the displacement of structure due to walking load motion.

These parameters of each term can then be elaborated by applying several of the following theories:

2.1 Single Lumped Mass

In practical works, engineers often tend to formulate the structural problems, especially dynamics ones, into simple structures which can be idealized for more convenient analysis. Obviously, the forces vertically subjected to the cantilever stairs are considered to be uniformly distributed forces, but it is then simplified into a simpler structure by having a concentrated or lumped mass m supported by a massless structure in the vertical direction.

Clearly, it is better to consider the distributed loading which are more likely to capture the true behavior of the structure. However, such idealization is claimed to be appropriate because it is still capable of maintaining some certain level of accuracy which reflects the behaviors and response of real structures as well as ensuring more safety by considering the worst case of having the load at the end of the cantilever stairs.

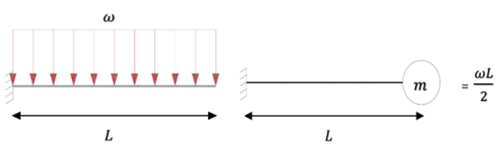

Fig. 1 Free body diagram of cantilever stairs

2.2 Force-Displacement Relation

Suppose there is a system which is subjected to an external static force fs with no dynamic excitation. Since the initial deformation is proportional to the applied force, the relationship between force fs and the displacement u is linear.

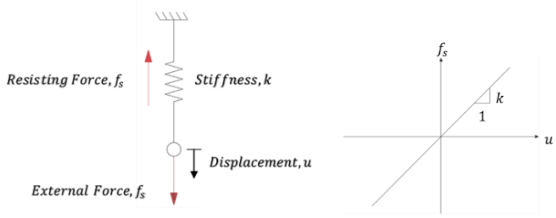

Fig. 2 Free body diagram of simple spring system

Therefore,

$$f_s = ku(t) \tag{2}$$

where k is the stiffness of the system.

2.3 Damping Force

Damping is the process that vibration steadily reduces in amplitude. In damping, the energy of the vibrating system is dissipated by various mechanisms. For real structures, there are often more than one mechanism that contribute to the energy dissipation. Because it is almost impossible to mathematically represent all of these energy dissipating mechanisms, the actual damping, especially for SDF structures, is usually idealized by a linear viscous damper. By assuming that the vibrational energy the damper dissipates is equivalent to the energy dissipated in all the damping mechanisms, the damping coefficient is then selected accordingly.

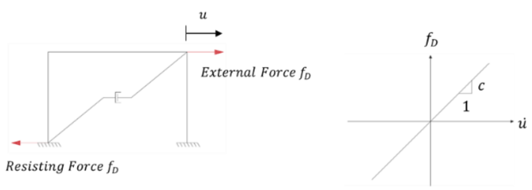

Fig. 3 Free body diagram of a damping System

$$f_D = c\dot{u}(t) \tag{3}$$

where $c = 2m\omega\xi$ is the viscous damping coefficient. m is the lumped mass, ω is the angular velocity, and ξ is the damping ration = 2%

2.4 Walking load on stair

Fourier series is a mathematic method to represent a periodic function as the summation of sine and cosine functions. In the study of human walking motion, the induced motion is shown as periodic movements of stepping and swinging if the movement is natural without any impact from external factor. Force that applied onto the surface of the structure during the stepping phase could be identified as the characteristic of double camel hump graph (shown as black line of Fig.4). Yet for simplifying the study, a new graph (red line of Fig.4) is given and used for numerical analysis with the studied magnitude of weight and same period as the stepping phase.

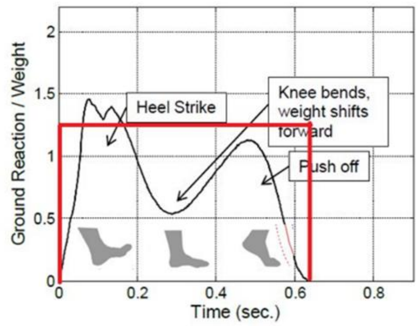

Fig. 4 Graph of walking load

Therefore, such motion can be express in term of the sine and cosine function using Fourier series method.

$$P(t) = W_p(C_0 + \sum_{n=1}^{4} r_n \sin(2n\pi f_p t + \emptyset_n)) \tag{4}$$

Where *P(t)* is the walking load varying on time, W_p is the weight of a single person = 80 kgf, *n* is the harmonic number of the Fourier series, r_n is the n^th Fourier coefficient, *t* is the time unit in millisecond, f_p is the walking frequency, and ϕ_n is the phase lag.

This equation, however, still requires a simple experiment to achieve two main controlling parameters which are the time period of the stepping phase, and that of the swinging phase.

Fig. 5 Experimental testing of a person ascending and descending a staircase

The graphs demonstrate the idealized loading of stepping phase and swinging phase of both ascending and descending directions in function of time. The motion presented as a continuous characteristic with the record point locating around the center of the stair height where the natural motion of human walking motion received least impact from the structure geometry. Since descending motion takes less time, its graph is also smaller than the ascending motion's.

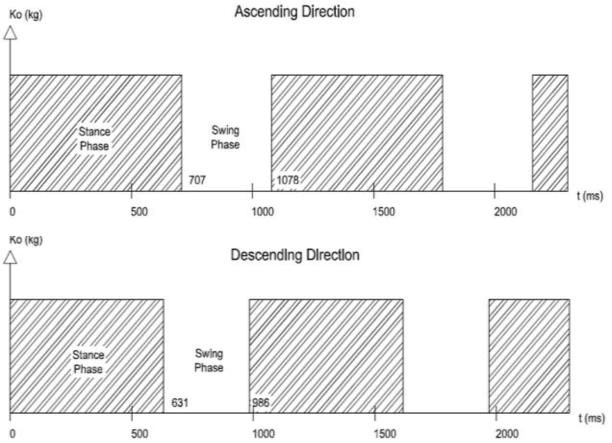

Fig. 6 Idealized loading of ascending and descending motions

However, both graphs are in the form of periodic function meaning that the walking cycles are considered to be identical by repeatedly maintaining the same time period all the times. In addition, force magnitudes decrease with increasing harmonic number and are insignificant beyond the third or fourth harmonic ([11] Murray TM et al). In order to convert the graph above into a Fourier series, 4th harmonic terms would be able to cover necessities of further analysis. As a result, it can be best written as a mathematic function by making the use of the Fourier series to create a function in form of sine and cosine function.

Table 1 Parameters of walking load of ascending and descending motions.

Parameter	Ascending				Descending			
	n=1	n=2	n=3	n=4	n=1	n=2	n=3	n=4
C_0	0.656				0.640			
r_n	0.56	0.26	0.02	0.15	0.58	0.25	0.05	0.16
f_p	0.927 Hz.				1.014 Hz.			
ϕ_n	-1.08	0.98	-0.10	-1.18	1.61	2.64	-0.26	1.65
Parameter	Ascending				Descending			
	n=1	n=2	n=3	n=4	n=1	n=2	n=3	n=4
C_0	0.656				0.640			
r_n	0.56	0.26	0.02	0.15	0.58	0.25	0.05	0.16
f_p	0.927 Hz.				1.014 Hz.			
ϕ_n	-1.08	0.98	-0.10	-1.18	1.61	2.64	-0.26	1.65

2.5 Virtual work method

The principle of virtual work method is used to formulate the deflection of the cantilever stairs system. The external virtual work done by the unit load 1.Δ which can be presented by below function:

$$\Delta = \int_0^L \frac{mM}{EI} dx \tag{5}$$

Where, Δ is displacement, ***m*** is virtual load, ***M*** is real load, ***E*** is elasticity of modulus of system and ***I*** is moment of inertia of stair.

The main actual goal is to determine the stiffness of the cantilever system by first obtaining the displacement of the system and converting it afterwards. The displacement at the end of the cantilever system is:

$$P = \frac{3EI}{L^3}\Delta \tag{6}$$

Where, ***P*** is applied load at the end of the cantilever stair.

Therefore, we can deduce that the stiffness of the cantilever system is:

$$k = \frac{3EI}{L^3} \tag{7}$$

where E is the elastic modulus of the system. I is the moment of inertia of stair, and L is span length of stair.

3. Methodology

The dimension of the RC. Cantilever stair used in this study is 1.0 meter in length, the width of the stair is 0.25 meters and the thickness is 0.10 meters. The details of the reinforcing steel in the cross-section of stairs are shown in Fig. 7

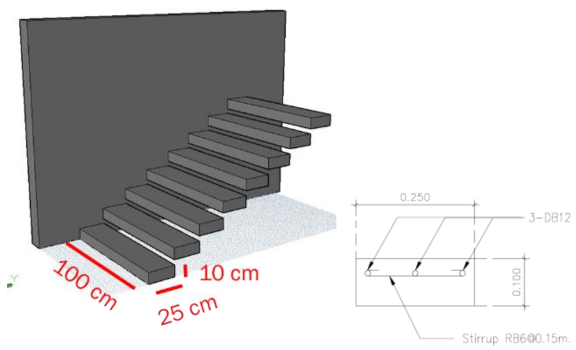

Fig. 7 Conceptual image of RC. cantilever stairs and its steel reinforcement

3.1 Reinforcing steel plates

The extra reinforcements which are used to lessen the dynamic effect of the RC cantilever stair system are steel plates with the yield strength of 4000 ksc and various dimensions as following:

- Flat bar 50mm x 4.5mm
- Flat bar 50mm x 6.0mm
- Flat bar 50mm x 8.0mm
- Flat bar 50mm x 9.0mm
- Flat bar 100mm x 6.0mm
- Flat bar 100mm x 8.0mm
- Flat bar 100mm x 9.0mm

The purpose of choosing the above sizes is to ensure that the size of the plate will not exceed the thickness of the thread of staircase which accounted for 10 cm when the bar is reinforced in lateral pattern. It is also ensured to allow at least 5 cm gaps between each reinforcement when installing on top surface pattern.

The position of the steel plate reinforcement toward the sample subject plays a major role in improving strength of the structure and reducing the vibration effect of the structure. Optimal positioning of the reinforcement will not only reduce the difficulties of installation, but also will increase the moment of inertia which will result more stable and stiffer staircase. In this study, there are two alternative choices of installation, whereas the plate will either be installed flat onto the top surface of the staircase or onto the side of the staircase (Lateral surface). The typical details for steel plate reinforcement are shown in Fig. 8

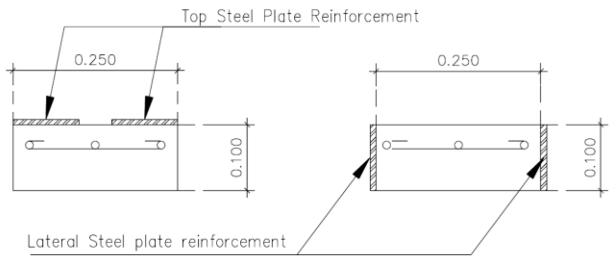

Fig. 8 Top and lateral steel plate reinforcement patterns.

3.2 Evaluations

The criteria that govern the design of the stairs system are strength and serviceability. Strength refers to the fact whether the structure can safely support and resist loadings or not, particularly shear stress and bending stress. The structure meets the strength criterion once the shear stress and bending stress induced by loadings do not exceed the capacity of the reinforced concrete design.

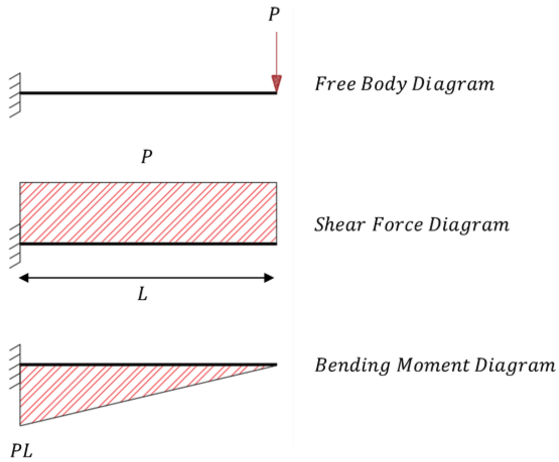

Fig. 9 Diagrams of cantilever beams

Serviceability, on the other hand, refers to deflection and dynamic response (acceleration) of the structure which are both dependent on stiffness. With the reinforcing steel plates, the structure's stiffness is expected to increase in order to have acceptable deformation as well as dynamic response under loading.

$$u \leq \frac{L}{360} \quad (8)$$

where u is dynamic deflection from the governing equation, L is span length of structural element.

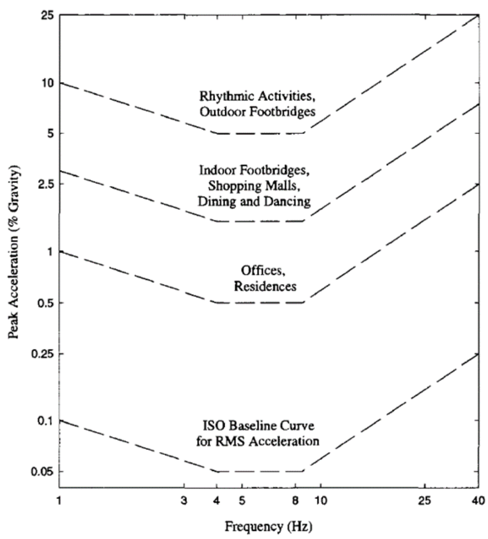

Fig. 10 Recommended peak acceleration for human comfort vibrations due to human activities

However, if the peak acceleration is chosen to be the governing parameter of the human perception of vibration occurring in the stair structure, it would be able to cover most of the discomfort felt by the users. The amount of time the system is subjected to this level of acceleration is too short for the human to actually perceive. Instead, the root-mean square acceleration is used.

$$a_{rms} = \frac{a_{peak}}{\sqrt{2}} \qquad (9)$$

4. Results and discussions

4.1 Properties of structures depending on Installation patterns of steel plates

In this section, all parameters necessary for the equation of motion are determined.

4.1.1 No Reinforcement

Table 2 Parameters for model without extra reinforcement

	m (kg) "Single Lumped Mass"	EI (N.m^2) "From Xtract"	k = 3EI/L^3 (kg/m)	c = 2mωξ "Damping Coefficient"
No Reinforcement	3.06	149400	45688	14.95

Table 3 Parameters for model with lateral reinforcement

Plate Sizes	m (kg) "Single Lumped Mass"	EI (N.m^2) "From Xtract"	k = 3EI/L^3 (kg/m)	c = 2mωξ "Damping Coefficient"
5cm 4.5mm	3.24	179,100	54,771	16.84
5cm 6mm	3.30	186,900	57,175	17.37
5cm 8mm	3.38	198,800	60,816	18.13
5cm 9mm	3.42	204,500	62,560	18.50
10cm 6mm	3.54	373,200	114,167	25.42
10cm 8mm	3.70	444,800	136,071	28.37
10cm 9mm	3.78	476,100	145,646	29.67

Table 4 Parameters for model with top reinforcement

Plate Sizes	m (kg) "Single Lumped Mass"	EI (N.m^2) "From Xtract"	k = 3EI/L^3 (kg/m)	c = 2mωξ "Damping Coefficient"
3@ 5cm 4.5mm	3.33	572,700	175,138	30.54
3@ 5cm 6mm	3.42	672,300	205,596	33.53
3@ 5cm 8mm	3.54	802,700	245,474	37.28
3@ 5cm 9mm	3.6	861,400	263,425	38.94
2@ 10cm 6mm	3.54	782,000	239,144	36.79
2@ 10cm 8mm	3.7	916,100	280,153	40.71
2@ 10cm 9mm	3.78	973,800	297,798	42.42

4.2 Analysis results

In this section, the analytical results in term of equation of motion for all installation pattern will be provided. The important results include the root mean square acceleration, the acceleration at the end of one cycle period of human motion, the maximum displacement. All of the results will be divided into ascending and descending motion type.

4.2.1 Lateral patterns

	Freq (Hz)	RMS a (m/s^2)	End Per. a (m/s^2)	Max. u (mm)	Vu (kg)	Mu (kg.m)
5cm 4.5mm	20.7	83.8 ✗	0.019 ✓	1.6 ✓	119.4 ✓	119.4 ✓
5cm 6mm	21.0	82.3 ✗	0.018 ✓	1.6 ✓	123.8 ✓	123.8 ✓
5cm 8mm	21.4	80.4 ✗	0.018 ✓	1.6 ✓	124.4 ✓	124.4 ✓
5cm 9mm	21.5	79.4 ✗	0.019 ✓	1.4 ✓	121.1 ✓	121.1 ✓
10cm 6mm	28.6	76.7 ✗	0.026 ✓	0.8 ✓	124.8 ✓	124.8 ✓
10cm 8mm	30.5	73.4 ✗	0.028 ✓	0.7 ✓	126.4 ✓	126.4 ✓
10cm 9mm	31.3	71.9 ✗	0.028 ✓	0.6 ✓	127.2 ✓	127.2 ✓

4.2.2 Top patterns

	Freq (Hz)	RMS a (m/s^2)	End Per. a (m/s^2)	Max. u (mm)	Vu (kg)	Mu (kg.m)
3@ 5cm 4.5mm	36.5	80.0 ✗	0.031 ✓	0.5 ✓	122.7 ✓	122.7 ✓
3@ 5cm 6mm	39.0	77.9 ✗	0.035 ✓	0.4 ✓	123.6 ✓	123.6 ✓
3@ 5cm 8mm	41.9	76.7 ✗	0.038 ✓	0.4 ✓	124.8 ✓	124.8 ✓
3@ 5cm 9mm	43.1	75.4 ✗	0.038 ✓	0.3 ✓	125.4 ✓	125.4 ✓
2@ 10cm 6mm	41.4	76.7 ✗	0.037 ✓	0.4 ✓	124.8 ✓	124.8 ✓
2@ 10cm 8mm	43.8	73.4 ✗	0.040 ✓	0.3 ✓	126.4 ✓	126.4 ✓
2@ 10cm 9mm	44.7	71.9 ✗	0.038 ✓	0.3 ✓	127.1 ✓	127.1 ✓

For the strength parameters, the results show that the loadings such as shear and moment which are obtained from the dynamic force due to human walking motion are all less than the capacity of the RC cantilever stairs can provide despite the ascending or descending direction.

Regarding the serviceability, both reinforced and non-reinforced structures satisfy the deflection parameter, meaning that the maximum value of the walking induced displacement is within the range of the acceptable deflection of the structure which is 2.7 mm. It is, however, a different case for the acceleration parameter. When checking the human perception of vibration, it is found that the RMS acceleration, which is the 70% of the peak acceleration and occurs in the transient state, is too high that it does not pass the design criteria proposed by the standard for all reinforcement patterns. Since the transient state of the vibration happens so fast within five milliseconds, the users might not be able to feel the vibrational acceleration yet. On the other hand, the acceleration drops abruptly to an acceptable value during the steady state. It is so small that even the non-reinforced structure passes the criteria. Therefore, it is difficult to make decision on which time phase, the amount of time that human can process the vibration information, should be considered.

For the installation, the steel plates reinforcing before or after construction does not make much difference at all. It can be easily observed that the outcome reduction in vibrational acceleration are quite similar to each other. What matters more is how the steel plates are reinforced. From the result, it is clearly that the top patterns lead to smaller acceleration, meaning that they are more effective in increasing the structure's stiffness and thus reducing the dynamic effects.

Last but not least, the result also shows that there is a big difference between the natural frequency of the structure and the frequency exerted from the human walking motion. The natural frequency of the structure is in the range of 20 to 50 Hz, whereas the force frequency is only 0.92 Hz for ascending motion, and 1.01 Hz for descending motion. It can be assured that the resonance disaster within the structure will never happen.

5. Conclusions

The presented study of the reinforcement of steel plates into RC cantilever stairs is mostly based on numerical analysis with the assistance of XTRACT, and MATLAB software. Several solutions and results have been found and described as below:

- The loading frequency of ascending and descending motions are 0.92 and 1.01 Hz respectively. This leads to no resonance disaster as the structure's is in the range of 20 to 50 Hz.
- It can be confirmed that there is no issue regarding the strength parameters, and the deflection of the structure.
- Top installation patterns prove to be more effective in reducing the dynamic effects.
- Accelerations in the steady state pass while those in the transient state don't. It is suggested that further research and experimentation should be done for more precise discovery.

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